Q. No. 2 (i) **GIVEN:** $\frac{x+1}{x} + \frac{x}{x+1} = \frac{25}{12}$

TO FIND: Solution of equation

METHOD: "Factorization"

SOLUTION:

Multiplying each term with the LCM of denominator

LCM: $12x(x+1)$

$12\cancel{x}(x+1)\left(\frac{x+1}{\cancel{x}}\right) + 12x\cancel{(x+1)}\left(\frac{x}{\cancel{x+1}}\right) = \cancel{12}x(x+1)\left(\frac{25}{\cancel{12}}\right)$

$12(x+1)^2 + 12x^2 = 25x(x+1)$ $\quad \because (a+b)^2 = a^2+2ab+b^2$

$12(x^2+2x+1) + 12x^2 = 25x^2 + 25x$

$12x^2 + 24x + 12 + 12x^2 = 25x^2 + 25x$

$24x^2 + 24x + 12 = 25x^2 + 25x$

$0 = 25x^2 - 24x^2 + 25x - 24x - 12$

$x^2 + x - 12 = 0$ (Standard form; $ax^2+bx+c=0$)

BY MIDTERM BREAK

$x^2 + 4x - 3x - 12 = 0$ $\quad (\because ac = -12)$

$x(x+4) - 3(x+4) = 0$ $\quad (-12 = +4x - 3)$

$(x-3)(x+4) = 0$

Either $x - 3 = 0$ or $x + 4 = 0$

$\boxed{x = 3}$, $\boxed{x = -4}$

Sol. set $= \{-4, 3\}$

Q. No. 2 (ii) GIVEN: $5^{1+x} + 5^{1-x} = 10$

TO FIND: Solution of equation

SOLUTION:

$5^{1+x} + 5^{1-x} = 10$

$5 \cdot 5^{x} + 5 \cdot 5^{-x} - 10 = 0$ $\quad \therefore a^{m+n} = a^{m} \cdot a^{n}$

$5(5^{x} + 5^{-x} - 2) = 0$

$5^{x} + \frac{1}{5^{x}} - 2 = 0$ $\quad \therefore a^{-n} = \frac{1}{a^{n}}$

Taking LCM,

$(5^{x})5^{x} + 1 - 2(5^{x}) = 0$

$5^{2x} - 2 \cdot 5^{x} + 1 = 0$ — (i)

Let $y = 5^{x}$ — (ii)

Squaring both sides of eq (ii)

$(y)^{2} = (5^{x})^{2}$

$y^{2} = 5^{2x}$

Equation (i) becomes,

$y^{2} - 2y + 1 = 0$ $\quad \therefore (ax^{2} + bx + c = 0)$

By midterm break,

$y^{2} - y - y + 1 = 0$

$y(y-1) - 1(y-1) = 0$

$(y-1)(y-1) = 0$

Either $y - 1 = 0$ or $y - 1 = 0$

$\boxed{y = 1}$, $\boxed{y = 1}$

• value of y is 1.

Replacing value of y in eq (ii)

$y = 5^{x}$

$5^{x} = 1$

$\rightarrow 5^{x} = 5^{0}$ $\quad \therefore a^{0} = 1$

When bases are same powers can be equated.

Q. No. 2 (iii) **GIVEN:** $x^2+(mx+c)^2=a^2$

TO SHOW: The equation has equal roots (i.e **disc = 0**)

CONDITION: $c^2=a^2(1+m^2)$

SOLUTION:

$x^2+(mx+c)^2=a^2$

$x^2+(mx)^2+(c)^2+2(mx)(c)=a^2 \quad \because (a+b)^2=a^2+b^2+2ab$

$x^2+m^2x^2+c^2+2mxc=a^2$

$x^2+m^2x^2+2mxc+c^2-a^2=0$

$(1+m^2)x^2+(2mc)x+(c^2-a^2)=0$

Comparing with, $Ax^2+Bx+C=0$

Here, $A=(1+m^2)$, $B=2mc$, $C=(c^2-a^2)$

Discriminant = B^2-4AC

Putting values,

$\text{Disc} = (2mc)^2-4(1+m^2)(c^2-a^2)$

$= 4m^2c^2-4(c^2-a^2+m^2c^2-m^2a^2)$

$= \cancel{4m^2c^2}-4c^2+4a^2-\cancel{4m^2c^2}+4m^2a^2$

$= -4c^2+4a^2+4m^2a^2$

Putting value of $c = a^2(1+m^2)$

$\text{Disc} = -4[a^2(1+m^2)]+4a^2+4m^2a^2$

$= -4(a^2+a^2m^2)+4a^2+4m^2a^2$

$= -\cancel{4a^2}-\cancel{4a^2m^2}+\cancel{4a^2}+\cancel{4m^2a^2}$

$\boxed{\text{Disc} = 0}$

As disc = 0, this means that the equation has equal roots because if **$b^2-4ac=0$, then the roots are rational (real) and <u>equal</u>.**

Hence proved!

Q. No. 2 (iv) **GIVEN:** w varies inversely as z.

$w = 5, \ z = 7$

TO FIND: a) Equation connecting w and z

b) value of constant

c) value of w when $z = 175/4$

SOLUTION:

As w varies inversely as z,

$$w \propto \frac{1}{z}$$

a) Converting into equality, $\boxed{w = \frac{K}{z}}$ — (i)

Putting values of w, z,

$K = wz$

$K = (5)(7)$

b) $\boxed{K = 35}$

Eq (i) becomes, $\boxed{w = \frac{35}{z}}$ — (ii)

c) **VALUE OF w WHEN z = 175/4:**

Putting $z = \frac{175}{4}$ in eq (ii)

$$w = \frac{35}{175/4}$$

$$w = 35 \div \frac{175}{4} \Rightarrow 35 \times \frac{4}{175}$$

$$\boxed{w = \frac{4}{5} \ (0.8)}$$

RESULT: • Equation connecting w and z is $\boxed{w = \frac{K}{z} \left(w = \frac{35}{z} \right)}$

• value of K is $\boxed{35}$

Q. No. 2 (v) **GIVEN:** $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$

TO PROVE: $\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3xyz}{abc}$

SOLUTION:

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3xyz}{abc}$$

AS, $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$

USING K-METHOD:

$$\frac{a}{x} = \frac{b}{y} = \frac{c}{z} = K \Rightarrow \frac{a}{x} = K, \frac{b}{y} = K, \frac{c}{z} = K$$

$\boxed{a = Kx}$, $\boxed{b = Ky}$, $\boxed{c = Kz}$

- **Taking L·H·S:** $\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3}$

Putting values of a, b, c

$$= \frac{x^3}{(Kx)^3} + \frac{y^3}{(Ky)^3} + \frac{z^3}{(Kz)^3}$$

$$= \frac{\cancel{x^3}}{K^3\cancel{x^3}} + \frac{\cancel{y^3}}{K^3\cancel{y^3}} + \frac{\cancel{z^3}}{K^3\cancel{z^3}} \Rightarrow \frac{1}{K^3} + \frac{1}{K^3} + \frac{1}{K^3} \Rightarrow \boxed{\frac{3}{K^3}} \quad \text{—(i)}$$

- **Taking R·H·S:** $\frac{3xyz}{abc}$

Putting values of a, b, c

$$= \frac{3xyz}{(Kx)(Ky)(Kz)} \Rightarrow \frac{3\cancel{xyz}}{K^3(\cancel{xyz})}$$

$$= \boxed{\frac{3}{K^3}} \quad \text{—(ii)}$$

From eq (i) and (ii), **L·H·S = R·H·S**

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3xyz}{abc}$$

Hence proved!

Q. No. 2 (vi)

Q. No. 2 (vii) **GIVEN :** $U = W$

$U = \{0, 1, 2, 3, 4, \dots\}$

$A = \{\ \}, \phi$

$B = \{1, 2, 3, 4, \dots\}$

TO FIND :

a) A'

b) B'

c) $(A \cup B)' = A' \cap B'$

SOLUTION :

a) $\boxed{A' = U - A}$

$= \{0, 1, 2, 3, \dots\} - \{\ \}$

$= \{0, 1, 2, 3, \dots\}$

b) $\boxed{B' = U - B}$

$= \{0, 1, 2, 3, \dots\} - \{1, 2, 3, \dots\}$

$= \{0\}$

c) $(A \cup B)' = A' \cap B'$

- **TAKING L.H.S :** $(A \cup B)'$

$(A \cup B) = \{\ \} \cup \{1, 2, 3, \dots\}$

$= \{1, 2, 3, \dots\}$

$(A \cup B)' = U - (A \cup B)$

$= \{0, 1, 2, 3, \dots\} - \{1, 2, 3, \dots\}$

$= \boxed{\{0\}}$ ——— (i)

- **TAKING R.H.S :** $A' \cap B'$

$A' = \{0, 1, 2, 3, 4, \dots\}$

$B' = \{0\}$

$A' \cap B' = \{0, 1, 2, \dots\} \cap \{0\}$

$= \boxed{\{0\}}$ ——— (ii)

From (i) and (ii), L.H.S = R.H.S

$(A \cup B)' = A' \cap B'$ **Hence proved**

(verified)

Q. No. 2 (viii) **GIVEN:** $X = \{x \mid x \in N \wedge x < 6\}$

$Y = \{y \mid y \in P \wedge y < 11\}$

TO FIND: a) X and Y in tabular form

b) $X \times Y$

c) $R = \{(x, y) \mid x + y = 6\}$

SOLUTION:

a) **IN TABULAR FORM:**

$X = \{1, 2, 3, 4, 5\}$

$Y = \{2, 3, 5, 7\}$

b) **$X \times Y$:**

Number of elements in $X \times Y = m \times n$

$= 5 \times 4$

$= \boxed{20}$

$X \times Y = \{1, 2, 3, 4, 5\} \times \{2, 3, 5, 7\}$

$= \{(1,2), (1,3), (1,5), (1,7),$
$(2,2), (2,3), (2,5), (2,7),$
$(3,2), (3,3), (3,5), (3,7),$
$(4,2), (4,3), (4,5), (4,7),$
$(5,2), (5,3), (5,5), (5,7)\}$

c) **RELATION $R = \{(x, y) \mid x + y = 6\}$:**

$R = \{(1,5), (3,3), (4,2)\}$

Dom $R = \{1, 3, 4\}$

Rang $R = \{2, 3, 5\}$

Q. No. 2 (ix) **GIVEN:**

Class limits	Frequency
4-6	10
7-9	20
10-12	13
13-15	7

TO FIND: a) Σf

b) $\Sigma f \log x$

c) Geometric Mean

SOLUTION:

CLASS LIMITS	f	x(Midpoint)	log x	f log x
4-6	10	5	0.6990	6.99
7-9	20	8	0.9031	18.062
10-12	13	11	1.0414	13.5382
13-15	7	14	1.1461	8.0227
Total:	50			46.6129

a) $\Sigma f = 50$

b) $\Sigma b \log x = 6.99 + 18.062 + 13.5382 + 8.0227$

$= 46.6129$

c) Geometric Mean: $\text{Antilog}\left(\frac{\Sigma f \log x}{\Sigma f}\right)$

$= \text{Antilog}\left(\frac{46.6129}{50}\right)$

$\boxed{G.M = \cancel{7.878}}$

$G.M = 8.556$

Q. No. 2 (x) **TO VERIFY:** $(\tan\theta + \cot\theta)(\cos\theta + \sin\theta) = \sec\theta + \operatorname{cosec}\theta$

SOLUTION:

$(\tan\theta + \cot\theta)(\cos\theta + \sin\theta) = \sec\theta + \operatorname{cosec}\theta$ — (i)

TAKING L.H.S:

$= (\tan\theta + \cot\theta)(\cos\theta + \sin\theta)$ $\quad \because \tan\theta = \dfrac{\sin\theta}{\cos\theta}, \cot\theta = \dfrac{1}{\tan\theta}$

$= \left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)\left(\cos\theta + \sin\theta\right)$

Taking LCM,

$= \left[\dfrac{\sin^2\theta + \cos^2\theta}{(\cos\theta)(\sin\theta)}\right](\cos\theta + \sin\theta)$

$= \dfrac{1}{(\cos\theta)(\sin\theta)} \times \cos\theta + \sin\theta$ $\quad \because \sin^2 + \cos^2\theta = 1$

$= \dfrac{\cos\theta + \sin\theta}{(\cos\theta)(\sin\theta)}$

$= \dfrac{\cancel{\cos\theta}}{(\cancel{\cos\theta})(\sin\theta)} + \dfrac{\cancel{\sin\theta}}{(\cos\theta)(\cancel{\sin\theta})}$

$= \dfrac{1}{\sin\theta} + \dfrac{1}{\cos\theta}$

$= \operatorname{cosec}\theta + \sec\theta$ $\quad \because \sec\theta = 1/\cos\theta, \operatorname{cosec}\theta = 1/\sin\theta$

$= \sec\theta + \operatorname{cosec}\theta$ — (ii)

From eq (i), (ii),

L.H.S = R.H.S

$(\tan\theta + \cot\theta)(\sin\theta + \cos\theta) = \sec\theta + \operatorname{cosec}\theta$

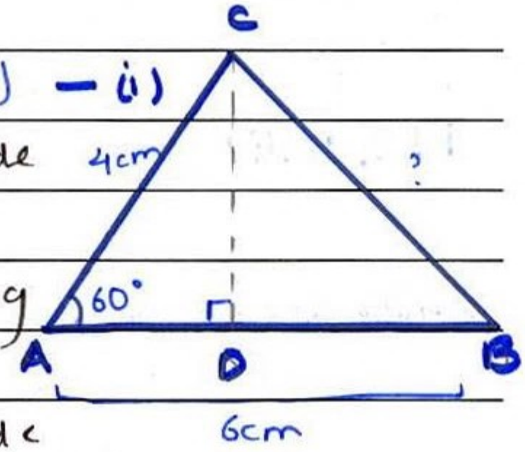
Hence proved!

Q. No. 2 (xi) **GIVEN:** $m\overline{AB} = 6cm$, $m\overline{AC} = 4cm$, $m\angle A = 60^\circ$

TO FIND: $m\overline{BC} = ?$

THEOREM: $(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD})$

SOLUTION:



$(m\overline{BC})^2 = (m\overline{AC})^2 + (m\overline{AB})^2 - 2(m\overline{AB})(m\overline{AD})$ — (i)

"In any triangle, the square of the side opposite to acute angle is equal to the sum of squares of sides containing the angle diminished by twice the rectangle contained by one of the side and the projection on it of the other."

In ∠rt $\triangle ADC$, $\cos\theta = \frac{\text{base}}{\text{hypotenuse}} \Rightarrow \cos\theta = \frac{m\overline{AD}}{m\overline{AC}}$

$$\text{base} = \cos 60^\circ \times 4cm$$

$$= \frac{4}{2} = 2cm$$

$$\boxed{m\overline{AD} = 2cm}$$

Putting values in eq (i)

$(m\overline{BC})^2 = (4)^2 + (6)^2 - 2(6)(2)$

$= 16 + 36 - 24$

$(m\overline{BC})^2 = 28$

Taking square root,

$\sqrt{(m\overline{BC})^2} = \sqrt{28}$

$\boxed{m\overline{BC} = 2\sqrt{7}\ cm}$

RESULT:

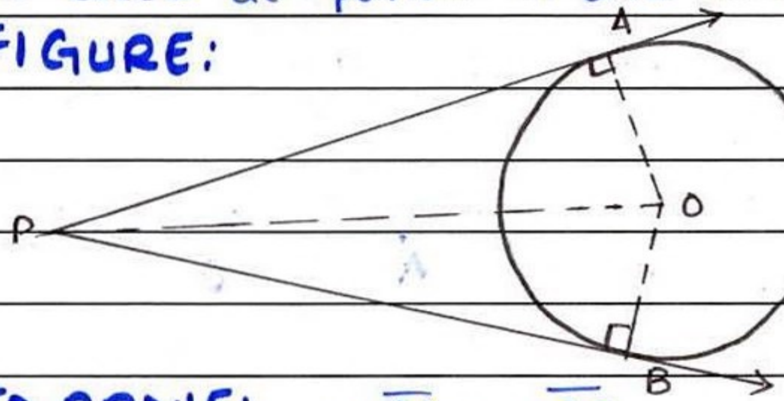
Length of $\overline{BC}$ is $2\sqrt{7}$ cm.

*CONSTRUCTION: Draw $\overline{CD} \perp AB$.

Q. No. 2 (xii) **STATEMENT:** Two tangents drawn to a circle from a point outside it are equal in length.

GIVEN: Two tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are drawn from a point P outside the circle with centre O which meet the circle at points A and B respectively.

FIGURE:



TO PROVE: $m\overline{PA} = m\overline{PB}$

CONSTRUCTION: Draw $\overline{OA} \perp \overline{PA}$ and $\overline{OB} \perp \overline{PB}$. Join O to P

PROOF:

STATEMENT	REASONS
In $\angle$rt Δs	
$\Delta OAP \leftrightarrow \Delta OBP$	
$m\overline{OP} = m\overline{OP}$	common
$m\angle OAP = m\angle OBP = 90^\circ$	Construction
$m\overline{OA} = m\overline{OB}$	radii of same circle
$\Delta OAP \cong OPBP$	**H.S postulate**
$m\overline{PA} = m\overline{PB}$	corresponding sides of congruent Δs.

RESULT: Hence proved that two tangents drawn to a circle from a point outside it are equal in length.

Q. No. 2 (xiii) **GIVEN :** $m\overline{AM} = m\overline{BM}$

$m\overline{OA} = 13$, $m\overline{OM} = 5$

TO FIND: a) value of $m\overline{BM} = ?$

b) $m\angle BOM = ?$

SOLUTION :

In rt $\angle$ Δ OMA, by pythagorus theorem

(hypotenuse)^2 = (base)^2 + (perpendicular)^2

$(m\overline{OA})^2 = (m\overline{AM})^2 + (m\overline{OM})^2$

$(13)^2 = (m\overline{AM})^2 + (5)^2$

$(m\overline{AM})^2 = 144$

$\sqrt{(m\overline{AM})^2} = \sqrt{144}$ $\quad$ ∴ Taking square root

$m\overline{AM} = \pm 12$

$m\overline{AM} = 12$ $\quad$ – i (length is always +ve)

AS $m\overline{AM} = m\overline{BM}$ $\quad$ – ii

$\boxed{mBM = 12}$ $\quad$ ∴ From (i) and (ii)

Q. No. 2 (xiv)

Q. No. 3 (Page 1/2)

GIVEN: Sum of squares of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits.

TO FIND: The number

SOLUTION:

let the 2 digits of number be x and y

let the number be,

$$xy = 10x + y$$

A.O.C

- $\boxed{x^2 + y^2 = 65}$ — (i)
- $10x + y = 9(x + y)$

$$10x + y = 9x + 9y$$
$$10x - 9x = 9y - y$$
$$\boxed{x = 8y} \quad \text{— (ii)}$$

BY SUBSTITUTION METHOD,

Putting value of x in eq (i)

$$(8y)^2 + y^2 = 65$$
$$64y^2 + y^2 = 65$$
$$65y^2 = 65$$
$$y^2 = 65/65$$
$$y^2 = 1$$

Taking square root,

$$\sqrt{y^2} = \sqrt{1}$$
$$y = \pm 1 \quad \text{(ignore -ve)}$$
$$\boxed{y = 1}$$

Put $y = 1$ in eq (ii)

$$x = 8(1)$$
$$\boxed{x = 8}$$

Q. No. 3 (Page 2/2)

The required number is $= 10x + y$

$= 10(8) + 1$

$= 80 + 1$

$= 81$

RESULT:

Required number is 81.

Q. No. 4 (Page 1/2)

GIVEN: $\dfrac{4x^2}{(1-x)(1+x^2)^2}$

TO FIND: Partial fractions

SOLUTION:

Resolving into partial fractions,

$$\frac{4x^2}{(1-x)(1+x^2)^2} = \frac{A}{1-x} + \frac{Bx+C}{(1+x^2)} + \frac{Dx+E}{(1+x^2)^2} \quad \text{— i}$$

Multiplying both sides with LCM, $(1-x)(1+x^2)^2$

$$4x^2 = A(1+x^2)^2 + (Bx+C)(1-x)(1+x^2) + (Dx+E)(1-x) \quad \text{— ii}$$

$$4x^2 = A(1+x^4+2x^2) + (Bx+C)(1+x^2-x-x^3) + Dx - Dx^2 + E - Ex$$

$$4x^2 = A + Ax^4 + 2Ax^2 + Bx + Bx^3 + Bx^2 - Bx^4 + Dx - Dx^2 + E - Ex$$

$$4x^2 = Ax^4 - Bx^4 + Bx^3 + 2Ax^2 - Bx^2 - Dx^2 + Bx - Ex + Dx + A + E$$

$4x^2 = (A-B)x^4 + (B$ [illegible] $x^3) + (2A-B-D)x^2 + (B-E+D)x + (A+E)$ — (iii)

BY ZERO'S METHOD,

Put $(1-x)=0 \Rightarrow x=1$ in eq (ii)

$$4(1)^2 = A(1+1)^2 + [B(1)+C][1-1)(1+1) + [D(1)+E](1-1)$$

$$4 = A(2)^2 + [C+B(0)] + (D+E(0))$$

$$4 = A(4)$$

$$\boxed{A = 1}$$

COMPARING COEFFICIENTS:

Q. No. 4 (Page 2/2) From eq (iii)

x^4 : $0 = A - B$ — iv

x^3 : $0 = B$ — v

x^2 : $4 = 2A - B - D$ — vi

x : $0 = B - E + D$ — vii

from eq iv,

$0 = A - B$

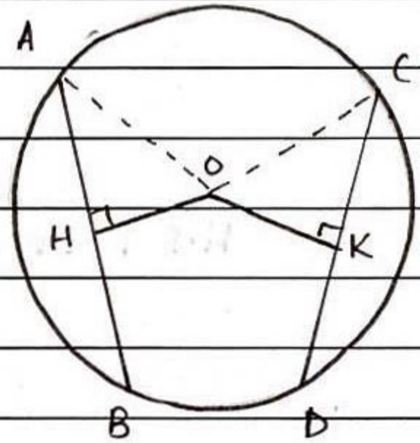
Q. No. 5 (Page 1/2)

Q. No. 5 (Page 2/2)

Q. No. 6 (Page 1/2)

STATEMENT: If two chords of a circle are congruent, then they will be equidistant from the centre.

FIGURE:



GIVEN: Two equal chords $\overline{AB}$ and $\overline{CD}$ in a circle with centre O such that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$.

TO PROVE: Chords are equidistant from centre i.e

$$m\overline{OH} = m\overline{OK}$$

CONSTRUCTION: Join O to A and C to form $\angle$rt Δs ΔOHA and ΔOKC

PROOF:

STATEMENT	REASONS
$\overline{AB}$ is the chord and $\overline{OH}$ bisects $\overline{AB}$	$\overline{OH} \perp \overline{AB}$ (~~const~~ given)
$m\overline{AH} = \frac{1}{2}(m\overline{AB})$ — (i)	
Similarly, $\overline{CD}$ is the chord and $\overline{OK}$ bisects it.	$\overline{OK} \perp \overline{CD}$
$m\overline{CK} = \frac{1}{2}(m\overline{CD})$ — (ii)	
But $m\overline{AB} = m\overline{CD}$ — (iii)	Given
So, $m\overline{AH} = m\overline{CK}$	From (i), (ii), (iii)

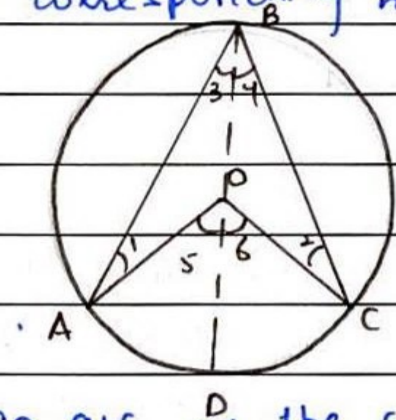
In $\angle rt\ \Delta s$,	
$\Delta OHA \leftrightarrow \Delta OKC$	
$m\angle OHA = m\angle OKC = 90^\circ$	Given ($\overline{OH} \perp \overline{AB}$) ($\overline{OK} \perp \overline{CD}$)
$m\overline{OA} = m\overline{OC}$	radii of same circle
$m\overline{AH} = m\overline{CK}$	Already proved
$\Delta OHA \cong \Delta OKC$	**H.S postulate**
$m\overline{OH} = m\overline{OK}$	Corresponding sides of congruent Δs.

RESULT: Hence proved that if two chords of a circle are congruent, then they will be equidistant from centre

Q. No. 7 (Page 1/2)

STATEMENT: The measure of the central angle of minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

FIGURE:



GIVEN: $\widehat{ABC}$ is an arc of the circle with centre O such that $\angle AOC$ is the central angle and $\angle ABC$ is the circum angle.

TO PROVE: $m\angle AOC = 2(m\angle ABC)$

CONSTRUCTION: Join O to B and produce it to D. Name the angles $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6$.

PROOF:

STATEMENT	REASON
In ΔAOB,	
• $m\angle 1 = m\angle 3$ (i)	Opposite angles of congruent sides
In ΔBOC,	
• $m\angle 2 = m\angle 4$ (ii)	Opposite angles of congruent sides
In ΔAOB,	
$m\angle 5 = m\angle 1 + m\angle 3$ (iii)	External angle of a triangle is equal to the sum of opposite interior angles
$m\angle 5 = m\angle 3 + m\angle 3 = 2m\angle 3$ (v)	From (i) and (iii)
In ΔBOC,	
$m\angle 6 = m\angle 2 + m\angle 4$ —(iv)	External angle is equal to sum of opposite internal angles.

$m\angle 5 + m\angle 6 = 2m\angle 3 + 2m\angle 4$	Adding eq (v) and (vi)
$m\angle 5 + m\angle 6 = 2(m\angle 3 + m\angle 4)$	
$\boxed{m\angle AOC = 2(m\angle ABC)}$	

RESULT: Hence proved that the measure of central angle of minor arc of circle is double than the angle subtended by corresponding major arc.

Graph Page No. 1

Graph Page No. 2

Rough Work 1

Rough Work 2